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Network Size and Connectivity in Mobile and Stationary Ad Hoc Networks

Lance Joneckis
Corinne Kramer
David Sparrow
David Tate

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INSTITUTE FOR DEFENSE ANALYSES
4850 Mark Center Drive
Alexandria, Virginia 22311-1882

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Network size and connectivity in mobile and stationary ad hoc networks

Lance Joneckis, Corinne Kramer, David Sparrow, and David Tate
Institute for Defense Analyses
4850 Mark Center Drive
Alexandria, VA USA 22311
dsparrow@ida.org

***Abstract*—One of the assumptions behind tactical mobile ad hoc networks (MANETs) is that routes consisting of multiple hops will be available to connect those nodes that lack line-of-sight connectivity. The practical utility of MANET depends on this assumption being true and, for a mobile network, remaining true during mobility. In previous work, we showed that average node-to-node direct connectivity and the rate at which direct links break over time while moving through a given terrain can be characterized approximately using average terrain measures that are location and distance independent. Here, we exploit that result and extend our previous work to predict dynamic network behavior for networks of various sizes in various terrains.**

Keywords—mobile ad hoc networks; telecommunications network reliability; terrain factors

I. INTRODUCTION

This paper investigates how mobility will affect network connectivity for terrains that have differing link-state probabilities. This investigation extends previous work [1] in which we examined the impact that three very different terrains would have on static network performance. The most direct impact of the terrain is allowing or blocking line-of-sight between node pairs in the network. We used the existence of line-of-sight between node pairs as a surrogate for link-state. We showed in a companion paper [2] that some aggregate features of networks can be captured in terms of average values of the link-state between node pairs. This finding allows an Erdős-Rényi (E-R) graph treatment [3] of network performance. In this paper, we exploit the fact that the distance scale for changes in link-state under mobility is also well described by an average value, which allows for a treatment analogous to detailed balance, as applied to chemical reactions in equilibrium, to describe the impact of node mobility on network features. In particular, we examine the rates at which pathways in a routing table will break as nodes move.

II. APPROACH

We selected a circular area approximately 16 km in diameter for each of the three regions. We used digital terrain maps [4] with horizontal spacing of approximately 30 m and vertical resolution of 1 m. Fig. 1 shows topographic maps of the three areas based on data from reference [4], with north-south and east-west elevation profiles. The three terrain sites chosen were Charlottesville, Virginia (CVA), White Sands, New Mexico (WNM), and Ft. Huachuca Arizona (HAZ). The latter sites were chosen because they are primary sites for ongoing testing of military ad-hoc networking radios. CVA was chosen because it represents a common rolling terrain that is quite different from the terrain found in the desert southwest.

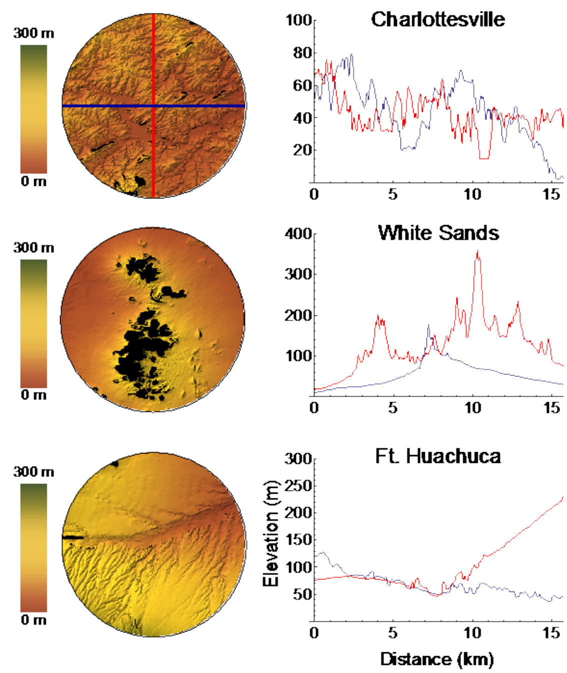

Fig. 1. Topological maps of the three areas. Elevation profiles across these areas are shown in red for north-south and blue for east-west.

CVA has a smaller range of elevations, but the spatial frequency of the topographic structure is greater—typical of rolling hills. The WNM site has a large irregular obstruction that will often block connectivity. This obstruction is impassible by vehicles, so we prohibit nodes from being placed at these locations (blacked area in Fig. 1). HAZ has a concave

structure in the north-south direction, which will tend to increase line-of-sight availability.

A companion paper at this conference [3] examines how accurately static performance can be captured in terms of the terrain-wide average link-state probability, *p*. In that paper, we calculated *p* for the three locations using a Monte Carlo simulation. Not surprisingly, aggregate features are qualitatively well described; however, features depending on individual node locations are not. In this current paper, we look at the impact of mobility on networks of size 10 and 20 nodes and for average link-state probabilities of 0.1, 0.2, and 0.4. For node separations between 2 and 8 km, these values of *p* are characteristic of the behavior at CVA, WSM, and HAZ, respectively. This fact gives us a set of examples that range from networks in which the expected number of links is smaller than the number of nodes to E-R networks in which the probability of forming a single connected network is high.

The average values of *p* for these locations are useful because the actual line-of-sight probabilities change relatively slowly with node location and separation. Thus, *p* can be treated as a quasi-static parameter, exhibiting "steady-state" behavior. The impact of mobility on communications performance will still depend upon how particular link-states change as nodes move and how this change disrupts a fixed network routing table.

Lambda (λ), the link-state correlation length, is the characteristic distance that a node will have to be displaced for the link to have a significant probability (one e-folding, or 37%) of changing state. We determined this characteristic length scale using a Monte Carlo simulation. Initially, the nodes were randomly laid across the terrain, and the link-state and separation, *d*, were noted. One of the nodes was displaced a random vector distance, with the magnitude restricted to less than $d/2$. The link-state was then recalculated. We binned the resulting link-states by *d* and plotted the percentage of each link transition (open-open, closed-closed, open-closed, closed-open) as a function of the displacement. From these data, we estimated λ at each of the binned values of *d* and have displayed these values in Fig. 2. The robustness of this approach was ensured with two checks: (1) we determined that the uniform sampling of magnitude and direction of the displacement did not influence the value of λ, and (2) we independently estimated the value of λ using the 0-1 and 1-0 transition data and confirmed that the values were the same.

For CVA and HAZ, we find nearly constant values of λ for initial separations between 2 and 8 km. Even for WNM, with the irregular obstruction dividing the region, the values of λ only vary about ±20%. Since most internode distances in a random laydown will fall within this 2- to 8-km band, these are the values of λ most relevant for modeling network behavior.

The slow variation of *p* and λ with separation means that the "steady state" to which we referred previously is a reasonable description: link probabilities are only mildly dependent on separation, particularly at operationally relevant separations, and node pairs change link-state when moving over distances that are also insensitive to initial separation. These characteristics allow application of E-R and detailed balance methods to evaluate mobile ad hoc network performance.

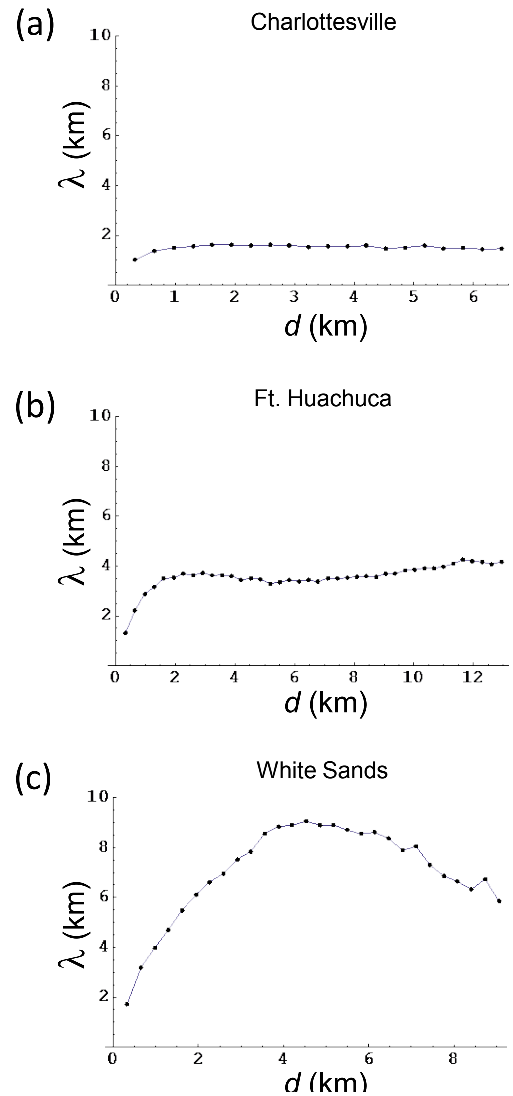

Fig. 2. The length scale over which link-states change as a function of initial node spearation. These values show little variation between 2 and 8 km for CVA or HAZ. Even for the rather extreme case of WNM, with the irregular obstacle in the center of the site, the length scale only varies by ±20%.

Table 1 shows the values of λ we used for the three locations.

TABLE I. CHARACTERISTIC LENGTH SCALE, λ, FOR THE THREE SITES

Location	**Characteristic Length Scale, λ**
CVA	1.5 km
WNM	7 km
HAZ	4 km

III. CALCULATONS

A. Determination of Expected Initial Network States

Given an E-R random network with N nodes and uniform link probability, p, we can compute the probability that a random pair of nodes (i,j) will be connected through some minimum number of hops $P_{ij}(n_{hops})$. For n_{hops} equal to one or two, we have closed form expressions in terms of p and N:

$$P_{ij}(1) = p \tag{1}$$

and

$$P_{ij}(2) = (1-p)(1-(1-p^2)^{N-2}). \tag{2}$$

We are unaware of any closed form expression for $n_{hops} > 2$, but we have derived a recursion relationship, the details of which will be presented in another forum, that allows us to enumerate $P_{ij}(n_{hops})$ given p and N. In our numerical results, we use this recursive solution, along with an assumption of "open shortest path first" (OSPF) in characterizing the routing table. Paths of different hop count within the routing table will break down and reconnect at different rates, depending upon the number of hops involved. We now turn to calculating the rate at which complete paths of k hops open, for $0 < k < N$.

B. Impact of Mobility on the Probability That an OSPF Path between Two Nodes is Preserved

A companion paper [3] shows that aggregate features of static network performance can be captured by descriptions based on an E-R model of connectivity using an average value for p. Using this E-R model is equivalent to assuming a steady-state condition for the probability of closed and open links that will be preserved under node mobility. Individual links will change state, but the behavior of the population will not. If λ is also relatively invariant, then detailed balance applies, with λ as the single rate constant. The solution to the evolution equation in this case is well known. If two nodes i and j are initially directly connected, then the evolution as a function of the displacement x is given by the factor

$$P_{evolve}(x) = \exp(-x/\lambda) + p(1-\exp(-x/\lambda)). \tag{3}$$

The combined probability that two nodes were initially directly connected and remain closed after displacement x is obtained by multiplying $P_{ij}(1)$ (which is just p) by the evolution factor in (3). In general, the nodes i and j can be connected through shortest paths, which will have path length between 1 and N–1 hops. The initial probability that i and j are connected by any number of hops, P_{ij}, is given by

$$P_{ij} = \Sigma_n P_{ij}(n), \tag{4}$$

and the probability that the (i,j) link is preserved after a displacement x is given by

$$P_{ij\ preserved}(x) = \Sigma_n P_{ij}(n)[\exp(-x/\lambda) + p(1-\exp(-x/\lambda))]. \tag{5}$$

The emphasis of this work is on how terrain will affect MANETs when nodes move. It does not incorporate any specifics about network management or routing beyond assuming a fixed routing table, based on an OSPF algorithm applied to the initial node configuration. In particular, these calculations do not include the possibility of the routing table being dynamically updated on a time scale that is short compared to the impact of mobility. Including these effects, in general, would require detailed knowledge of the algorithms and apply only to that approach.

Before turning to specific results, we note a few general behaviors. First, the slope of $P_{ij\ preserved}(x)$ scales like $(1-p)/\lambda$, so links break more quickly given either short λ or low p. Further, the probability that a node pair remains connected will fall more rapidly when the shortest paths have more hops, which is also true when p is low.

IV. RESULTS

In this section, we present the probability that an (i,j) pair remains connected when the nodes move, as a function of node displacement, in kilometers. We examined six cases: 10- and 20-node networks for each of the three locations. Fig. 3 shows the probability that a node pair is initially connected (displacement zero) and remains connected as displacement increases.

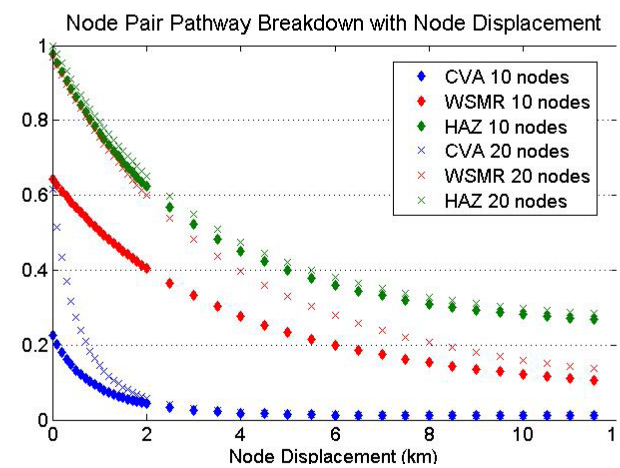

Fig. 3. The probability of a shortest path connecting a random pair of nodes remaining connected as the nodes move. The probability is plotted as a function of the node displacement, for networks with 10 and 20 nodes, with link probabilites (0.1, 0.2, and 0.4) characteristic of CVA, WNM and HAZ, respectively. Initial values less than 1.0 reflect the chance that no path exists between the nodes in the original laydown.

We note first that the probability of the shortest path being preserved falls to a common level nearly independent of network size for each terrain. For the highest p region, HAZ, the behavior is essentially independent of network size for all displacements because nearly all shortest paths in the initial laydown comprise only one or two hops. Note that, in all cases, the probability of an original shortest path being preserved falls below the probability of having a direct link. In other words, if routing table updates cannot be made quickly enough, a routing algorithm of "connect directly, or wait until you can" would outperform a self-healing ad hoc protocol. For CVA, the convergence of the 20-node network $P_{ij\ preserved}$ values to those

of the 10-node network is especially rapid. The terrain's short characteristic length and low *p* lead to longer initial pathways, which have a lower probability of being preserved. (The average path length for 10 nodes in CVA is less than 2.0, but for 20 nodes is just over 3.0.) These effects combine to force the rapid convergence.

Fig. 3 shows that a 20-node laydown in CVA has the same path existence probabilities as a 10-node network in WNM. However, the behavior once the nodes begin to move is very different. Table II compares the probability of a path between two nodes, the expected number of links that are used in some shortest path (where such paths exist), and the rate at which links in these paths break under mobility. We see that the price of matching the WSM probability of connection by increasing the network size at CVA is a factor of almost 6 in the number of links that must be maintained and a factor of more than 25 in the rate at which routing links change state. Thus, while it may be possible to overcome a low link probability by adding nodes to a static network, this solution would impose a significant burden on network management during mobility.

TABLE II. PROBABIITY OF LINK CLOSURE FOR THE THREE SITES

Location and Network Size	P_{ij} – The Probability Two Nodes Are Initially Connected	Expected Number of Links in Some Shortest Path	Rate of Change of Links/ Kilometer
CVA 10 nodes	0.225	19	13
CVA 20 nodes	0.615	360	240
WNM 10 nodes	0.643	64	9
WNM 20 nodes	0.969	415	59
HAZ 10 nodes	0.978	76	19
HAZ 20 nodes	0.9999	309	77

Finally, we note that the addition of an air tier—unless it is a privileged node with unlimited capacity—does little to mitigate the effects of mobility. Adding an airborne node after the terrestrial nodes are placed can be used to improve the network by tying together subnets or reaching isolated nodes. However, once mobility begins, the link-states between the terrestrial nodes change. Paths that originally linked directly through the air tier will be maintained, while all others have high probability of being lost. Updates to the routing table during mobility may be sufficient to reroute the traffic, but the demand on the air tier will tax the capacity of a single node and will obviate the need for an ad hoc networking scheme.

V. SUMMARY AND CONCLUSIONS

We have examined the link-state between pairs of nodes for three regions that have very different terrain features. The average probability of link closure and the displacement scale over which the link-state changes do not depend strongly on node separation [1]. The relatively slow variation in the probability of link closure allows for an Erdős-Rényi (E-R) graph treatment of the initial state of a network [2]. In this work, we exploit the slow variation in the length scale for link-state changes to model how the pathways between nodes will evolve as the nodes move.

We find that the probability of initial routing paths being preserved falls rapidly with node displacement (on the order of 2 km) for all three regions examined. The probability that the pathway will remain successful converges to a terrain-specific constant for networks of 10 or more nodes. In all cases, the probability that the original paths will remain connected falls below the probability of a direct connection between the nodes.

For a static network, the probability that two nodes will be connected can always be increased by adding nodes since there is some probability that an added node will close a pathway that did not exist before and there is no chance that the added node will break existing paths. However, we find that the network management burden increases dramatically with network size in some terrains. Adding nodes in these terrains is unlikely to provide a useful solution for a maneuvering network.

Adding an airborne node can, in principle, ensure that paths always exist among all terrestrial nodes, even under mobility. However, this solution does not lead to a successful implementation of MANET. If the airborne node has high capacity, the network will quickly converge to a static topology with all traffic routed through the airborne node, making the ad hoc capabilities of the terrestrial nodes irrelevant. If the airborne node has limited capacity, it will either fail to provide reliable connectivity among terrestrial nodes or (worse) be unable to support the offered load it receives.

For the cases at which we have looked, both the static and the mobile behaviors of networks are strongly influenced by terrain. When assessing possible network performance and in designing approaches to network management, explicit treatment of terrain impacts should be incorporated in future operational concepts. Except for the most benign environments, MANETs will need to be terrain-aware to deliver high performance.

ACKNOWLEDGMENT

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Corinne Kramer
David Sparrow
David Tate

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14. ABSTRACT

One of the assumptions behind tactical mobile ad hoc networks (MANETs) is that routes consisting of multiple hops will be available to connect those nodes that lack line-of-sight connectivity. The practical utility of MANET depends on this assumption being true and, for a mobile network, remaining true during mobility. In previous work, we showed that average node-to-node direct connectivity and the rate at which direct links break over time while moving through a given terrain can be characterized approximately using average terrain measures that are location and distance independent. Here, we exploit that result and extend our previous work to predict dynamic network behavior for networks of various sizes in various terrains.

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